Abstracts — Triangle Area Graduate Mathematics Conference
Saturday, March 21, 2015

Keynote
9:00 Ezra Miller, Applying persistent homology to brain artery and vein imaging

Persistent homology measures geometric structures using topological invariants. When the structures are magnetic resonance images of branching arteries, for example, persistent homology records the connectedness of an increasing subset of the vessels. Although the theory of persistent homology is relatively well developed, and many aspects of its behavior are understood in synthetic examples, only recently have applications to genuine experimental data begun. This talk explains what we have learned about the geometry of blood vessels in aging human brains, as well as lessons this exploration has taught us about applications of persistent homology in general. These lessons inform further developing applications of persistent homology in statistical problems from biological and medical imaging. The main results are joint with Paul Bendich, Steve Marron, Aaron Pieloch, and Sean Skwerer (at the time, a Math postdoc, Stat faculty, Math undergrad, and Operations Research grad student).

Session 1
10:20 Margaret Rahmoeller (mlrahmoe@ncsu.edu), Demazure Crystals for \( U_q \left( A_n^{(1)} \right) \)

The theory of crystal base provides a combinatorial approach to studying representation theory of quantum affine Lie algebras. Demazure modules are certain finite dimensional subspaces of integrable highest weight modules generated by extremal vectors. In this talk we discuss the representation theory of the affine special linear Lie algebra and use examples to describe the Demazure crystals for the quantum affine Lie algebra \( U_q \left( A_n^{(1)} \right) \).

10:50 Alex Combs (ancombs2@ncsu.edu), The BKP hierarchy as a twisted module over a lattice vertex algebra

The BKP hierarchy of partial differential equations was first discussed by Jimbo, Kashiwara, and Miwa in their series of papers on transformation groups for soliton equations. In the papers, they construct the BKP hierarchy using a vertex operator similar to the vertex operator construction of the KP hierarchy. We discuss a construction of the type B boson-fermion correspondence and the BKP hierarchy from a twisted module of a lattice vertex algebra corresponding to the lattice \( \mathbb{Z} \) and discuss an extension to \( \mathbb{Z}^n \).

11:20 David Lax* (dclax@live.unc.edu), Robert Proctor, Explicit Bases for Embedding Coordinates of Schubert Varieties via Scanning Paths

Schubert varieties are certain sets of flags in a vector space. They can be defined simply using matrices and rank conditions on minors. Young tableaux label coordinates for a projective embedding of Schubert varieties. We want an explicit basis for its embedding coordinates. A basis was first given in the 1980s. In this talk we present a new basis theorem using Scanning Tableaux, a combinatorial tool developed in 2012.

11:50 Catherine Maria Hsu (cmhsu@live.unc.edu), Two Classes of Number Fields with a Non-Principal Euclidean Ideal

In 1979, Lenstra defined the Euclidean ideal, a generalization of the Euclidean algorithm. Just as the existence of a Euclidean algorithm for the ring of integers \( \mathcal{O}_K \) in a number field \( K \) implies
a trivial class group, the existence of a Euclidean ideal $C$ in $\mathcal{O}_K$ implies a cyclic class group with generator $[C]$. By using certain growth results, Graves provided the first explicit example of a number field a non-principal Euclidean ideal. In this talk, we generalize Graves’ techniques in order to introduce two classes of totally real quartic number fields, one of biquadratic extensions and one of cyclic extensions, each of which has a non-principal Euclidean ideal.

Session 2

10:20 Quinn Alexander Morris* (qamorris@uncg.edu), R. Shivaji, R. Dhanya, Existence of positive radial solutions for superlinear, semipositone problems on the exterior of a ball

We study positive radial solutions to $-\Delta u = \lambda K(|x|) f(u); \ x \in \Omega_\varepsilon$ where $\lambda > 0$ is a parameter, $\Omega_\varepsilon = \{ x \in \mathbb{R}^N : |x| > r_0, r_0 > 0, N > 2 \}$, $\Delta$ is the Laplacian operator, $K \in C([-r_0, \infty), (0, \infty))$ satisfies $K(r) \leq \frac{1}{r^{N+\mu}}; \ \mu > 0$ for $r \gg 1$, and $f \in C^1([0, \infty), \mathbb{R})$ is a class of non-decreasing functions satisfying $\lim_{s \to \infty} \frac{f(s)}{s} = \infty$ (superlinear) and $f(0) < 0$ (semipositone). We will be interested in solutions, $u$, such that $u \to 0$ as $|x| \to \infty$, and which also satisfy the nonlinear boundary condition $\frac{\partial u}{\partial \eta} + \tilde{c}(u)u = 0$ when $|x| = r_0$, where $\frac{\partial}{\partial \eta}$ is the outward normal derivative, and $\tilde{c} \in C([0, \infty), (0, \infty))$. We will establish the existence of a positive radial solution for small values of the parameter $\lambda$. We also establish a similar result for the case when $u$ satisfies the Dirichlet boundary condition ($u = 0$) for $|x| = r_0$. We establish our results via variational methods, namely using the Mountain Pass Lemma.

10:50 Michael Benfield* (mike.benfield@gmail.com), Irina Kogan, Kris Jenssen, Quadratically Interacting Systems

Carefully constructed conservation laws whose solutions blow up have been used to explore the ill-posedness of these systems for large data. Here, we examine the possibility of constructing systems whose waves interact quadratically.

11:20 Byungjae Son* (b_son@uncg.edu), Ratnasingham Shivaji, Bifurcation and multiplicity results for classes of $p,q$ Laplacian systems

We study positive solutions to boundary value problems of the form:

$$
\begin{align*}
-\Delta_p u &= \lambda \{u^{p-1-\alpha} + f(v)\} \quad \text{in } \Omega, \\
-\Delta_q v &= \lambda \{v^{q-1-\beta} + g(u)\} \quad \text{in } \Omega, \\
u &= 0 \quad \text{on } \partial \Omega,
\end{align*}
$$

where $\Delta_m u := \text{div}(|\nabla u|^{m-2} \nabla u); \ m > 1$ is the $m$-Laplacian operator of $u$, $\lambda > 0$, $p > 1$, $q > 1$, $\alpha \in (0, p-1)$, $\beta \in (0, q-1)$ and $\Omega$ is a bounded domain in $\mathbb{R}^N; N \geq 1$ with smooth boundary $\partial \Omega$. Here $f,g : [0, \infty) \to \mathbb{R}$ are nondecreasing $C^1$ functions with $f(0) = g(0) = 0$. We first establish that for $\lambda \approx 0$ there exist positive solutions bifurcating from the trivial branch ($\lambda, u \equiv 0, v \equiv 0$) at $(0,0,0)$. We further discuss an existence result for all $\lambda > 0$ and a multiplicity result for a certain range of $\lambda$ under additional assumptions of $f$ and $g$. We employ the method of sub-super solutions to establish our results.

10:50 Amarjit Budhiraja, Ruoyu Wu* (wuruoyu@live.unc.edu), Some Fluctuation Results for Weakly Interacting Multi-type Particle System

A weakly interacting multi-type particle system is considered. The dynamics is given in terms of a collection of diffusion equations with each particle’s initial condition governed independently by a probability law that depends only on its type. The system models a network of $N$
interacting spiking neurons and has been previously studied by Faugeras et al. (2012) where the authors study the law of large numbers behavior and prove a propagation of chaos result. In our work we study fluctuations about the law of large numbers limit and establish a central limit theorem. We also consider a modification of the model where neuron of each type is influenced by a common noise source. Fluctuations in this case are characterized, instead of by a Gaussian limit, through a mixture of Gaussian distributions. Limit theorems for symmetric statistics of such weakly interacting particles, given in terms of multiple stochastic integrals, are also established.

Session 3

10:20 George Bernard Lankford* (gblankfo@ncsu.edu), Hien Tran, Michael Read, Lawrence Ives, Kelsey Reppert, Kayla Cline, Juan Guzman, Optimization of Klystron Designs using Deterministic Sampling Methods

In this talk, we present a method for optimizing the design of klystron circuits. This automates the selection of cavity positions, resonant frequencies, quality factors, R/Q and other circuit parameters to maximize the efficiency with required gain. The method is based on deterministic sampling methods. In this talk we describe the procedure and give several examples for both narrow and wide band klystrons, using the klystron codes AJDISK and TESLA.

10:50 Cagatay Karan (ckaran@ncsu.edu), A Black Litterman Model for CVaR Optimization

The Black Litterman Model (BLM) has contributed to modern portfolio theory a new perspective where the investor views and market equilibrium expected excess returns are combined in a Bayesian manner to get the optimal portfolio weights. Bertsimas, Gupta and Paschalidis (2012) have showed that one can get BLM type results by using inverse optimization. We will show our algorithms and numerical results for the BLM type optimization problems under CVaR risk measure.

11:20 Emese Kennedy (ealipcse@ncsu.edu), Swing-up and Stabilization of an Inverted Pendulum: Real-Time Implementation

The single inverted pendulum is a classic example of a nonlinear system. It is considered as one of the most popular benchmarks of nonlinear control theory. Many nonlinear methods have been proposed for the swing-up and stabilization of a self-erecting inverted pendulum, however, most of these techniques are too complex and impractical for real-time implementation. During the presentation we will discuss the successful real-time implementation of a nonlinear controller for balancing an inverted pendulum based on the power series approximation to the Hamilton Jacobi Bellman equation. We will also discuss the implementation of an energy based control method for the swing-up of the pendulum.

Session 4

10:20 Daniel Irving Bernstein* (dibernst@ncsu.edu), Seth Sullivant, Unimodular Binary Hierarchical Models

Hierarchical models are an important class of statistical models that can be studied using combinatorics and algebraic geometry. If a hierarchical model is binary, then we can totally describe it with a simplicial complex. A hierarchical model is called unimodular if a certain related integer programming problem is always equivalent to its linear relaxation. Our main result is a complete classification of the simplicial complexes that give rise to unimodular binary
hierarchical models. We give a characterization in terms of forbidden minors, as well as in more constructive terms. We also suggest some ways that one might extend this classification to all hierarchical models.

10:50 **Colby Long*** (celong2@ncsu.edu), **Seth Sullivant**, *Tying up loose strands: the defining equations of the strand symmetric model*

The strand symmetric model is a phylogenetic model designed to reflect the symmetry inherent in the double-stranded structure of DNA. We show that the set of known phylogenetic invariants for the general strand symmetric model of the three leaf claw tree entirely defines the ideal. This knowledge allows one to determine the vanishing ideal of the general strand symmetric model of any trivalent tree. Our proof of the main result is computational. We use the fact that the Zariski closure of the strand symmetric model is the secant variety of a toric variety to compute the dimension of the variety. We then show that the known equations generate a prime ideal of the correct dimension using elimination theory.

11:20 **Tulay Ayyildiz Akoglu*** (tayyild@gmail.com), **A. Szanto, J. D. Hauenstein**, *Certifying Solutions to overdetermined and singular systems over Q*

Given an overdetermined system with m polynomials with n variables over Q. For some m < n, and assume that the ideal generated by these m polynomials is zero dimensional. We will describe two methods to certify exact roots near numerical approximations. Novelty of our method is it can certify overdetermined system and singular roots as well. Both methods are hybrid symbolic-numeric methods.

11:50 **Joey Hart*** (jlhart3@ncsu.edu), **Pierre Gremaud**, *Variable Importance with Random Forest Models*

Variable importance using Random Forests is an area active research across several disciplines. The existing methods lack a clear mathematical definition and have very few theoretical results. In this talk we will propose a definition for variable importance in terms of optimal subsets. We subsequently present an algorithm to compute optimal subsets and explore preliminary numerical results.

Section 5

2:00 **Mary Ambrosino*** (meambros@ncsu.edu), **Hoon Hong, Eunjeong Lee**, *Maximum Gap in Inverse Cyclotomic Polynomials*

Cyclotomic polynomials are fundamental in number theory and have many applications in diverse areas, such as cryptography. When designing certain cryptosystems, it is crucial to know the sparsity structure of the inverse of the cyclotomic polynomial, that is, the maximum gap between two consecutive exponents that appear.

In this talk, we will present three results on the maximum gap of inverse cyclotomic polynomials.

1. We will provide a (lower and upper) bound.
2. We will provide an exact expression under a certain condition.
3. We will show that when the first prime is fixed, the condition almost always holds.

2:30 **Alaa Al-Kateeb*** (aqalkate@ncsu.edu), **Hoon Hong, Eunjeong Lee**, *Number of Terms in Cyclotomic Polynomials*

Cyclotomic polynomials play crucial roles in algebraic number theory with applications in various areas such as pairing over certain family of elliptic curves. The structure of the cyclotomic
polynomials, in particular the number of terms (Hamming weight), determine the efficiencies of the resulting pairings.

In this talk, we will present two curious results on the number of terms of cyclotomic polynomials of np-th order.

(1) Linearity: The number of terms for np is linear over p when p is the same modulo n. (2) Parallelism: The number of terms for np and np’ are parallel when p and -’p are the same modulo n.

These result can be used to determine efficiently the number of terms of the np-th cyclotomic polynomials for very large p, without actually computing the np-th cyclotomic polynomials.

This is joint work with Hoon Hong and Eunjeong Lee.

3:00 Emily Barnard* (esbarnar@ncsu.edu), Nathan Reading, biCatalan Combinatorics

We consider several counting problems related to Coxeter-Catalan combinatorics. Some of the problems are short on a priori motivation, but together they acquire a posteriori motivation from their answers. Specifically, we conjecture that the problems all have the same answer, and we prove the conjecture in many cases. We call the solution to these problems the W-biCatalan number, because the problems constitute a new instance of the mysterious enumerative coincidences at the core of Coxeter-Catalan combinatorics.

Session 6

2:00 Robert Bryant, Ingrid Daubechies, Tingran Gao* (trgao10@math.duke.edu), The Diffusion Geometry of Shape Spaces

We introduce the concept of Hypoelliptic Diffusion Maps (HDM), a framework generalizing Diffusion Maps in the context of manifold learning and dimensionality reduction. Standard non-linear dimensionality reduction methods (e.g., LLE, ISOMAP, Laplacian Eigenmaps, Diffusion Maps) focus on mining massive data sets using weighted affinity graphs; Orientable Diffusion Maps and Vector Diffusion Maps enrich these graphs by attaching to each node also some local geometry. HDM likewise considers a scenario where each node possesses additional structure, which is now itself of interest to investigate. Virtually, HDM augments the original data set with attached structures, and provides tools for studying and organizing the augmented ensemble. The goal is to obtain information on individual structures attached to the nodes and on the relationship between structures attached to nearby nodes, so as to study the underlying manifold from which the nodes are sampled. In this talk, we analyze HDM on tangent bundles, revealing its intimate connection with sub-Riemannian geometry and a family of hypoelliptic differential operators. The future work will focus on more general fibre bundles.

2:30 Phillip Andreae (pandreae@math.duke.edu), Metric (in)dependence of analytic torsion and generalizations

Analytic torsion is a number constructed from spectral data associated to a flat vector bundle over a Riemannian manifold \((M, g)\). Under appropriate conditions, it may be interpreted as a topological invariant of \(M\), i.e., it is independent of the metric \(g\). We frame the metric independence argument in terms of the closedness of a certain differential form on the space of metrics, and we define generalized "higher" analytic torsions and study their dependence on the metric. This is a preliminary report.
3:00 **Radmila Sazdanovic, Dan Scofield***(dscofie@ncsu.edu), *Torsion in chromatic graph homology*

Khovanov homology is an invariant of knots and links that categorifies the Jones polynomial. Helme-Guizon and Rong (2005) introduced a Khovanov-type, bigraded homology for graphs that categorifies the chromatic polynomial. The occurrence of torsion is of particular interest. We compute torsion over the algebra $A_2$ for a variety of graphs and present several conjectures.

3:30 **Harish Chintakunta, Jennifer Gamble***(jpgamble@ncsu.edu), Hamid Krim, *Emergence of Core-Periphery Structure from Local Node Dominance in Social Networks*

There has been growing evidence recently for the view that social networks can be divided into a well connected core, and a sparse periphery. This talk describes how such a global description can be obtained from local “dominance” relationships between vertices. It is shown that the resulting core describes the global structure of the network, while also preserving shortest paths, preserving homology of the flag complex of the network, and displaying “expander-like” properties. Moreover, the periphery obtained from this decomposition consists of a large number of connected components, which can be used to identify communities in the network. These are used for a ‘divide-and-conquer’ strategy for community detection, where the peripheral components are obtained as a pre-processing step to identify the small sets most likely to contain communities. The method is illustrated using a real world network (DBLP co-authorship network), with ground-truth communities.

**Session 7**

2:00 **Lucas Castle, Kristina Marie Martin***(martin.kristina@gmail.com), Daniel Toundykov, Jean-Paul Zolesio, *Optimal Control in a Free Boundary Fluid-Elasticity Interaction*

We consider an optimal control problem involving a free boundary fluid-elasticity interaction described by Navier-Stokes coupled with the equations of nonlinear elastodynamics. We prove that turbulence in the fluid flow can be minimized using a distributed control and discuss the first order necessary optimality conditions. This is work in progress in collaboration with Lorena Bociu, Lucas Castle (North Carolina State University), Daniel Toundykov (University of Nebraska, Lincoln), and Jean-Paul Zolesio (INRIA and CNRS-INLN, Sophia-Antipolis, France).

2:30 **Nicholas Anthony Battista***(bubbles@unc.edu), Andrea Lane, John Cruickshank, Laura Miller, *Hematocrit and trabeculation: it takes two to tango...or something like that*

Proper cardiogenesis requires a delicate balance between genetic and environmental (epigenetic) factors, and mechanical forces. Hemodynamic processes, such as vortex formation, are important in the generation of shear at the endothelial surface layer and strains at the epithelial layer, which aid in proper morphology and functionality. Hematocrit first appears in embryonic zebrafish hearts around 25 hpf, while ventricular trabeculae form around 72 hpf, for Womersley number (Wo) on the order of 0.1. Effects of hematocrit and trabeculation in this flow regime is not well understood. In this study, computational fluid dynamics is used to quantify the effects of Wo, idealized trabeculae morphology, and hematocrit on intracardial flows in embryonic zebrafish hearts.
Roberto Camassa, Zhi Lin, Richard McLaughlin, Keith Mertens, Chung-Nan Tzou*
(pgzt31@live.unc.edu), James Walsh, Brian White, Optimal mixing of buoyant jets and
plumes in stratified fluids: theory and experiments

The influence of ambient fluid stratification on buoyant miscible jets and plumes is studied the-
oretically and experimentally. Given a fixed set of jet/plume parameters, and an ambient fluid
stratification sandwiched between top and bottom homogenous densities, a theoretical criterion
is identified showing how step-like density profiles constitute the most effective mixers within a
broad class of stable density transitions. This is assessed both analytically and experimentally,
respectively by establishing rigorous a priori estimates on generalized Morton-Taylor-Turner
(MTT) (Morton et al. 1956; Fischer et al. 1979) models, and by studying a critical phe-
nomenon determined by the distance between the jet/plume release height with respect to the
depth of the ambient density transition. For fluid released sufficiently close to the background
density transition, the buoyant jet fluid escapes and rises indefinitely. For fluid released at
locations lower than a critical depth, the buoyant fluid stops rising and is trapped indefinitely.
A mathematical formulation providing rigorous estimates on MTT models is developed along
with nonlinear jump conditions and an exact critical-depth formula in good quantitative agree-
ment with the experiments. Our mathematical analysis provides rigorous justification for the
critical trapping/escaping criteria, first presented in Caulfied and Woods (1998), within a class
of algebraic density decay rates. Further, the analysis uncovers surprising differences between
the Gaussian and Top-hat profile closures concerning initial mixing of the jet and ambient fluid.

Steven Derochers (sjderoch@ncsu.edu), On the Semigroup Generator for the Total Lin-
earization of a Hydro - Elasticity Model

We investigate the semigroup generator for the total linearization of a hydrodynamic model
with respect to a forcing term. One of the key differences from the classical Stokes-elasticity
system is that the elastic component gives rise to an elliptic problem with an oblique derivative,
where the coefficients depend on the curvature of the fluid-structure interface. We examine
numerically and analytically the maximality of the evolution generator and the ellipticity of
the elastic sub-problem.

Session 8

Kristen Tillman (kristen_tillman@ncsu.edu), Model Comparison Tests and Data Infor-
mation Content in Inverse Problems: an Insect Population Dynamics Case Study

In the context of inverse problems, we examine a case study of population dynamics of a cotton-
feeding insect to demonstrate the use of statistically based model comparison tests. In this
example we are interested in questions related to information content of a given data set and
whether the data will support a more complicated model to describe it. We investigate whether
the information content in data sets for the pest Lygus hesperus as it is currently collected
is sufficient to support a model in which one distinguishes between juveniles and adults. In
addition we investigate whether a data set collected from pesticide-treated fields will support
a model using time dependent parameters.
Daphnia magna is a vital species in ongoing investigations into the synergistic toxicity of chemicals. They serve as an early biological warning system for an ecosystem, and are found in virtually all bodies of water. In this study we use data from individual and population level experiments we conducted in order to calibrate and validate a multi-scale age structured model to describe the population dynamics of D. magna.